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Quantum Mechanical Effects of a Nondissipative Mesoscopic Capacitance Coupling Circuit with Source

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The charge and current quantum zero-point fluctuations in a squeezed vacuum state of a nondissipative mesoscopic capacitance coupling circuit with source are given. The quantum noise of this circuit at absolute zero temperature is derived. It is found that there is a squeezing effect in this coupling circuit.

In recent years, rapid progress in nanometer techniques and nanoelectronics (Srivastava and Widom, 1987; Buot, 1993) has made it possible for the miniaturization of circuits and devices to reach atomic dimensions (Garcia, 1992). When the transport scale of electrons reaches their nonelastic impact scale, quantum effects of circuits and devices must be considered. Recently, much attention has been paid to the study of mesoscopic physics (Dekker, 1979; Chen *et al.*, 1995, 1996a, b; Li and Chen, 1996). The present paper studies the quantum mechanical effects in a nondissipative mesoscopic capacitance coupling circuit with source.

The classical Hamiltonian for a classical nondissipative capacitance coupling circuit with source is

$$H = \frac{P_1^2}{2L_1} + \frac{q_1^2}{2C_1} + \frac{(q_1 - q_2)^2}{2C_0} + \frac{P_2^2}{2L_2} + \frac{q_2^2}{2C_2} - q_1\mathscr{E}(t) \tag{1}$$

where L_i, C_i, C_0, and q_i ($i = 1, 2$) stand for inductance, capacitances, and charge, respectively, and $\mathscr{E}(t)$ is the electromotive force, a function of time.

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The quantities $P_i = L_i\, dq_i/dt$ and q_i $(i = 1, 2)$ are conjugate. If we let

$$[q_i, P_i] = i\hbar \tag{2}$$

then the quantization of equation (1) can be realized. Taking the transformations

$$\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} \left(\dfrac{L_2}{L_1}\right)^{1/4} & 0 \\ 0 & \left(\dfrac{L_1}{L_2}\right)^{1/4} \end{bmatrix} \begin{bmatrix} \cos\dfrac{\varphi}{2} & \sin\dfrac{\varphi}{2} \\ -\sin\dfrac{\varphi}{2} & \cos\dfrac{\varphi}{2} \end{bmatrix} \begin{bmatrix} q_1' \\ q_2' \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} \left(\dfrac{L_1}{L_2}\right)^{1/4} & 0 \\ 0 & \left(\dfrac{L_2}{L_1}\right)^{1/4} \end{bmatrix} \begin{bmatrix} \cos\dfrac{\varphi}{2} & \sin\dfrac{\varphi}{2} \\ -\sin\dfrac{\varphi}{2} & \cos\dfrac{\varphi}{2} \end{bmatrix} \begin{bmatrix} P_1' \\ P_2' \end{bmatrix} \tag{4}$$

we find that becomes

$$\begin{aligned} \hat{H} = {} & \frac{1}{2\sqrt{L_1L_2}}(p_1'^2 + p_2'^2) + \frac{\alpha}{2} q_1'^2 + \frac{\beta}{2} q_2'^2 + \gamma q_1' q_2' \\ & - \left(\frac{L_2}{L_1}\right)^{1/4}\left(q_1' \cos\frac{\varphi}{2} + q_2' \sin\frac{\varphi}{2}\right)\mathscr{E}(t) \end{aligned} \tag{5}$$

where

$$\alpha = \left(\frac{1}{C_0} + \frac{1}{C_1}\right)\sqrt{\frac{L_2}{L_1}}\cos^2\frac{\varphi}{2} + \left(\frac{1}{C_0} + \frac{1}{C_2}\right)\sqrt{\frac{L_1}{L_2}}\sin^2\frac{\varphi}{2} + \frac{\sin\varphi}{C_0} \tag{6}$$

$$\beta = \left(\frac{1}{C_0} + \frac{1}{C_1}\right)\sqrt{\frac{L_2}{L_1}}\sin^2\frac{\varphi}{2} + \left(\frac{1}{C_0} + \frac{1}{C_2}\right)\sqrt{\frac{L_1}{L_2}}\cos^2\frac{\varphi}{2} - \frac{\sin\varphi}{C_0} \tag{7}$$

$$\gamma = \left(\frac{1}{2C_0} + \frac{1}{2C_1}\right)\sqrt{\frac{L_2}{L_1}}\sin\varphi - \left(\frac{1}{2C_0} + \frac{1}{2C_2}\right)\sqrt{\frac{L_1}{L_2}}\sin\varphi - \frac{\cos\varphi}{C_0} \tag{8}$$

In order to eliminate the cross term in equation (5), let $\gamma = 0$; we have

$$\operatorname{tg}\varphi = \frac{2}{C_0}\left[\left(\frac{1}{C_0} + \frac{1}{C_1}\right)\sqrt{\frac{L_2}{L_1}} - \left(\frac{1}{C_0} + \frac{1}{C_2}\right)\sqrt{\frac{L_1}{L_2}}\right]^{-1} \tag{9}$$

Correspondingly, α and β can be determined by the angle ϕ.

Furthermore, we take

$$\xi = \left(\frac{L_2}{L_1}\right)^{1/4} \cos\frac{\varphi}{2}\mathscr{E}(t), \qquad \eta = \left(\frac{L_2}{L_1}\right)^{1/4} \sin\frac{\varphi}{2}\mathscr{E}(t) \tag{10}$$

Equation (5) becomes

$$\hat{H} = \frac{1}{2\sqrt{L_1L_2}}(P_1'^2 + P_2'^2) + \frac{\alpha}{2}\left(q_1' - \frac{\xi}{\alpha}\right)^2 + \frac{\beta}{2}\left(q_2' - \frac{\eta}{\beta}\right)^2 - \left(\frac{\xi^2}{2\alpha} + \frac{\eta^2}{2\beta}\right) \tag{11}$$

In terms of the creation and annihilation operators a_i^+ and a_i $(i = 1, 2)$, q_i' and p_i' $(i = 1, 2)$ can be represented as

$$q_1' = \frac{1}{\sqrt{2}\mu}(a_1 + a_1^+), \qquad P_1' = (-i)\frac{\hbar\mu}{\sqrt{2}}(a_1 - a_1^+) \tag{12}$$

$$q_2' = \frac{1}{\sqrt{2}\nu}(a_2 + a_2^+), \qquad P_2' = (-i)\frac{\hbar\nu}{\sqrt{2}}(a_2 - a_2^+) \tag{13}$$

where

$$[a_1, a_1^+] = [a_2, a_2^+] = 1 \tag{14}$$

$$\mu = \hbar^{-1/2}\alpha^{1/4}(L_1L_2)^{1/8}, \qquad \nu = \hbar^{-1/2}\beta^{1/4}(L_1L_2)^{1/8} \tag{15}$$

we have

$$\hat{H} = \frac{\alpha}{\mu^2}\left(a_1^+ - \frac{\mu\xi}{\sqrt{2}\alpha}\right)\left(a_1 - \frac{\mu\xi}{\sqrt{2}\alpha}\right) + \frac{\beta}{\nu^2}\left(a_2^+ - \frac{\nu\eta}{\sqrt{2}\beta}\right) \times \left(a_2 - \frac{\nu\eta}{\sqrt{2}\beta}\right) + \frac{\alpha}{2\mu^2} + \frac{\beta}{2\nu^2} - \left(\frac{\xi^2}{2\alpha} + \frac{\eta^2}{2\beta}\right) \tag{16}$$

We introduce new operators as follows:

$$\tilde{a}_1^+ = a_1^+ - \frac{\mu\xi}{\sqrt{2}\alpha}, \qquad \tilde{a}_1 = a_1 - \frac{\mu\xi}{\sqrt{2}\alpha} \tag{17}$$

$$\tilde{a}_2^+ = a_2 - \frac{\nu\eta}{\sqrt{2}\beta}, \qquad \tilde{a}_2 = a_2 - \frac{\nu\eta}{\sqrt{2}\beta} \tag{18}$$

They satisfy the commutation relations $[\tilde{a}_1, \tilde{a}_1^+] = [\tilde{a}_2, \tilde{a}_2^+] = 1$. Equation (16) becomes

$$\hat{H} = \frac{\alpha}{\mu^2}\left(\tilde{a}_1^+\tilde{a}_1 + \frac{1}{2}\right) + \frac{\beta}{\nu^2}\left(\tilde{a}_2^+\tilde{a}_2 + \frac{1}{2}\right) - \left(\frac{\xi^2}{2\alpha} + \frac{\eta^2}{2\beta}\right) \tag{19}$$

The eigenvalue of equation (19) is

$$E = \left(n_1 + \frac{1}{2}\right)\hbar w_1 + \left(n_2 + \frac{1}{2}\right)\hbar w_2 - \left[\frac{1}{2\alpha}\cos^2\frac{\varphi}{2}\mathscr{E}^2(t)\sqrt{\frac{L_2}{L_1}} + \frac{1}{2\beta}\sin^2\frac{\varphi}{2}\mathscr{E}^2(t)\sqrt{\frac{L_2}{L_1}}\right] \tag{20}$$

where

$$w_1 = \left(\frac{\alpha}{\sqrt{L_1L_2}}\right)^{1/2}, \qquad w_2 = \left(\frac{\beta}{\sqrt{L_1L_2}}\right)^{1/2} \tag{21}$$

From equations (16)–(18), we find the ground state of this circuit

$$|0, 0\rangle' = |0\rangle_1' \otimes |0\rangle_2' = \exp\left[\frac{\mu\xi}{\sqrt{2\alpha}}(a_1^+ - a_1)\right]|0\rangle_1 \otimes \exp\left[\frac{\nu\eta}{\sqrt{2\beta}}(a_2^+ - a_2)\right]|0\rangle_2 \tag{22}$$

We can define the vacuum state $|0, 0\rangle$ from $[a_1, a_1^+] = [a_2, a_2^+] = 1$. When the circuit has no power, we have $|0, 0\rangle' \to |0, 0\rangle$.

We introduce the squeezed vacuum state $|0, 0\rangle_{r_1,r_2}$; the formula for expressing this state takes the form (Fan and Guo, 1985)

$$|0, 0\rangle_{r_1,r_2} = \mathrm{sech}^{1/2}\, r_1 \sum_{n=0}^{\infty} \frac{(-\mathrm{th}\, r_1)^n(2n!)^{1/2}}{n!\, 2^n}|2n\rangle_1 \otimes \mathrm{sech}^{1/2}\, r_2 \sum_{n=0}^{\infty} \frac{(-\mathrm{th}\, r_2)^n(2n!)^{1/2}}{n!\, 2^n}|2n\rangle_2 \tag{23}$$

where r_1 and r_2 stand for squeezed parameters with $r_1, r_2 \in [0, \infty)$.

The squeezed vacuum state is only the superposition of an even number of states. In the squeezed vacuum states, the average and the mean-square values of q_i, q_i^2, p_i, and p_i^2 ($i = 1, 2$) are, respectively,

$$\langle q_1\rangle = {}_{r_1,r_2}\langle 0, 0|q_1|0, 0\rangle_{r_1,r_2} = 0, \qquad \langle q_2\rangle = {}_{r_1,r_2}\langle 0, 0|q_2|0, 0\rangle_{r_1,r_2} = 0 \tag{24}$$

$$\langle p_1\rangle = {}_{r_1,r_2}\langle 0, 0|P_1|0, 0\rangle_{r_1,r_2} = 0, \qquad \langle P_2\rangle = {}_{r_1,r_2}\langle 0, 0|P_2|0, 0\rangle_{r_1,r_2} = 0 \tag{25}$$

$$\langle q_1^2\rangle = {}_{r_1,r_2}\langle 0, 0|q_1^2|0, 0\rangle_{r_1,r_2} = \sqrt{\frac{L_2}{L_1}}\left[\frac{1}{2\mu^2}\cos^2\frac{\varphi}{2}\operatorname{sech} r_1 \times\left(\sum_{n=0}^{\infty}\frac{(-\operatorname{th} r_1)^n(2n!)^{1/2}}{n!\,2^n}\right)\left(\sum_{n=0}^{\infty}\frac{(-\operatorname{th} r_1)^n(2n!)^{1/2}(4n+1)}{n!\,2^n}\right) + \frac{1}{2\nu^2}\sin^2\frac{\varphi}{2}\operatorname{sech} r_2\left(\sum_{n=0}^{\infty}\frac{(-\operatorname{th} r_2)^n(2n!)^{1/2}}{n!\,2^n}\right) \times\left(\sum_{n=0}^{\infty}\frac{(-\operatorname{th} r_2)^n(2n!)^{1/2}(4n+1)}{n!\,2^n}\right)\right] \tag{26}$$

$$\langle q_2^2\rangle = {}_{r_1,r_2}\langle 0, 0|q_2^2|0, 0\rangle_{r_1,r_2} = \sqrt{\frac{L_1}{L_2}}\left[\frac{1}{2\mu^2}\sin^2\frac{\varphi}{2}\operatorname{sech} r_1 \times\left(\sum_{n=0}^{\infty}\frac{(-\operatorname{th} r_1)^n(2n!)^{1/2}}{n!\,2^n}\right)\left(\sum_{n=0}^{\infty}\frac{(-\operatorname{th} r_1)^n(2n!)^{1/2}(4n+1)}{n!\,2^n}\right) + \frac{1}{2\nu^2}\cos^2\frac{\varphi}{2}\operatorname{sech} r_2\left(\sum_{n=0}^{\infty}\frac{(-\operatorname{th} r_2)^n(2n!)^{1/2}}{n!\,2^n}\right) \times\left(\sum_{n=0}^{\infty}\frac{(-\operatorname{th} r_2)^n(2n!)^{1/2}(4n+1)}{n!\,2^n}\right)\right] \tag{27}$$

$$\langle P_1^2\rangle = {}_{r_1,r_2}\langle 0, 0|P_1^2|0, 0\rangle_{r_1,r_2} = \sqrt{\frac{L_1}{L_2}}\left[\frac{\hbar\mu^2}{2}\cos^2\frac{\varphi}{2}\operatorname{sech} r_1\left(\sum_{n=0}^{\infty}\frac{(-\operatorname{th} r_1)^n(2n!)^{1/2}}{n!\,2^n}\right) \times\left(\sum_{n=0}^{\infty}\frac{(-\operatorname{th} r_1)^n(2n!)^{1/2}(4n+1)}{n!\,2^n}\right) + \frac{\hbar^2\nu^2}{2}\sin^2\frac{\varphi}{2}\operatorname{sech} r_2 \times\left(\sum_{n=0}^{\infty}\frac{(-\operatorname{th} r_2)^n(2n!)^{1/2}}{n!\,2^n}\right)\left(\sum_{n=0}^{\infty}\frac{(-\operatorname{th} r_2)^n(2n!)^{1/2}(4n+1)}{n!\,2^n}\right)\right] \tag{28}$$

$$\langle P_2^2\rangle = {}_{r_1,r_2}\langle 0, 0|P_2^2|0, 0\rangle_{r_1,r_2} = \sqrt{\frac{L_2}{L_1}}\left[\frac{\hbar^2\mu^2}{2}\sin^2\frac{\varphi}{2}\operatorname{sech} r_1\left(\sum_{n=0}^{\infty}\frac{(-\operatorname{th} r_1)^n(2n!)^{1/2}}{n!\,2^n}\right) \times\left(\sum_{n=0}^{\infty}\frac{(-\operatorname{th} r_1)^n(2n!)^{1/2}(4n+1)}{n!\,2^n}\right) + \frac{\hbar^2\nu^2}{2}\cos^2\frac{\varphi}{2}\operatorname{sech} r_2 \times\left(\sum_{n=0}^{\infty}\frac{(-\operatorname{th} r_2)^n(2n!)^{1/2}}{n!\,2^n}\right)\left(\sum_{n=0}^{\infty}\frac{(-\operatorname{th} r_2)^n(2n!)^{1/2}(4n+1)}{n!\,2^n}\right)\right] \tag{29}$$

From equations (24)–(29), we find that the average values of q_i and p_i ($i = 1, 2$) are zero when the circuit has no power, but their mean-square values are not zero; the results indicate that the charge and current show zero-point fluctuations.

When the circuit has power, we have for the circuit ground state $|0,0\rangle'$ [see equation (22)]

$$\langle q_1\rangle = {}'\langle 0,0|q_1|0,0\rangle' = \left(\frac{L_2}{L_1}\right)^{1/4}\left(\frac{\xi}{\alpha}\cos\frac{\varphi}{2} + \frac{\eta}{\beta}\sin\frac{\varphi}{2}\right) \tag{30}$$

$$\langle q_2\rangle = {}'\langle 0,0|q_2|0,0\rangle' = \left(\frac{L_1}{L_2}\right)^{1/4}\left(\frac{-\xi}{\alpha}\sin\frac{\varphi}{2} + \frac{\eta}{\beta}\cos\frac{\varphi}{2}\right) \tag{31}$$

$$\langle P_1\rangle = {}'\langle 0,0|P_1|0,0\rangle' = 0, \qquad \langle P_2\rangle = {}'\langle 0,0|P_2|0,0\rangle' = 0 \tag{32}$$

$$\langle q_1^2\rangle = {}'\langle 0,0|q_1^2|0,0\rangle' = \sqrt{\frac{L_2}{L_1}}\left[\frac{1}{2\mu^2}\cos^2\frac{\varphi}{2}\left(1 + \frac{2\mu^2\xi^2}{\alpha^2}\right)\right.$$
$$\left. + \frac{1}{2v^2}\sin^2\frac{\varphi}{2}\left(1 + \frac{2v^2\eta^2}{\beta^2}\right) + \frac{\xi\eta}{\alpha\beta}\sin\varphi\right] \tag{33}$$

$$\langle q_2^2\rangle = {}'\langle 0,0|q_2^2|0,0\rangle' = \sqrt{\frac{L_1}{L_2}}\left[\frac{1}{2\mu^2}\sin^2\frac{\varphi}{2}\left(1 + \frac{2\mu^2\xi^2}{\alpha^2}\right)\right.$$
$$\left. + \frac{1}{2v^2}\cos^2\frac{\varphi}{2}\left(1 + \frac{2v^2\eta^2}{\beta^2}\right) - \frac{\xi\eta}{\alpha\beta}\sin\varphi\right] \tag{34}$$

$$\langle P_1^2\rangle = {}'\langle 0,0|P_1^2|0,0\rangle'$$
$$= \sqrt{\frac{L_1}{L_2}}\left[\frac{\hbar^2\mu^2}{2}\cos^2\frac{\varphi}{2} + \frac{\hbar^2v^2}{2}\sin^2\frac{\varphi}{2}\right] \tag{35}$$

$$\langle P_2^2\rangle = {}'\langle 0,0|P_2^2|0,0\rangle'$$
$$= \sqrt{\frac{L_2}{L_1}}\left[\frac{\hbar^2\mu^2}{2}\sin^2\frac{\varphi}{2} + \frac{\hbar^2v^2}{2}\cos^2\frac{\varphi}{2}\right] \tag{36}$$

In the above calculations of equations (30)–(36), we have used the Baker–Campbell–Hausdoff formula

$$e^ABe^{-A} = B + [A, B] + \frac{1}{2!}[A, [A, B]] + \cdots \tag{37}$$

From equations (30)–(36), we find that the average and the mean-square values of the charges (q_1 and q_2) are quite different depending on whether

$\epsilon(t)$ is switched on or off; on the other hand, those of the currents (p_1 and p_2) are the same whether $\epsilon(t)$ is switched on or off. The fluctuations of charges and currents are related to each other, so squeezing occurs in this coupling circuit.

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